

**Quadratic Equations**

# Introduction to Quadratic equation

If *p*(*x*) is a quadratic polynomial, then *p*(*x*) = 0 is called a **quadratic equation**.

The general or standard form of a quadratic equation, in the variable *x*, is given by *ax2 + bx + c* = 0, where *a, b, c* are real numbers and *a*  0.

# Roots of the quadratic equation

* + The value of *x* that satisfies an equation is called the **zeroes** or **roots** of the equation.
  + A real number  is said to be a solution/root of the quadratic equation *ax2 + bx + c* = 0 if *aα*2 + *b + c*

= 0.

* + A quadratic equation has **at most two roots**.

1. A quadratic equation can be solved by following algebraic methods:
2. Splitting the middle term (factorization)
3. Completing squares
4. Quadratic formula

# Splitting the middle term (or factorization) method

* + If *ax2 + bx + c, a*  0, can be reduced to the product of two linear factors, then the roots of the quadratic equation *ax2 + bx + c* = 0 can be found by equating each factor to zero.
  + Steps involved in solving quadratic equation factorization) method:

**Step 1:** Find the product *ac*.

*ax*2  *bx*  *c*  0 (*a*  0) by **splitting the middle term** (or

**Step 2:** Find the factors of ‘ac’ that add to up to *b,* using the following criteria:

1. If *ac*>0 and *b*>0, then both the factors are positive.
2. If *ac*>0 and *b*<0, then both the factors are negative.
3. If *ac*<0 and *b*>0, then larger factor is positive and smaller factor is negative.
4. If *ac*<0 and *b*<0, then larger factor is negative and smaller factor is positive.

**Step 3:** Split the middle term into two parts using the factors obtained in the above step.

**Step 4:** Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.

**Step 5:** Equate each of the linear factors to zero to get the value of x.

# Completing the square method

* + Any quadratic equation can be converted to the form (*x + a*)2 – *b*2 = 0 or (x – a)2 + b2 = 0 by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
  + The steps involved in solving a quadratic equation by **completing the square**, are as follows:

**Step 1:** Make the coefficient of x2 unity.

**Step 2:** Express the coefficient of x in the form 2 × *x* × *p*.

**Step 3:** Add and subtract the square of *p*.

**Step 4:** Use the square identity (*a + b*)2 or (*a – b*)2 to obtain the quadratic equation in the required form (*x + a*)2 - *b*2 = 0 or (x – a)2 + b2 = 0.

**Step 5:** Take the constant term to the other side of the equation.

**Step 6:** Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

# Quadratic formula

The roots of a quadratic equation *ax*2  *bx*  *c*  0 (*a*  0) can be calculated by using the **quadratic formula:**

*b*  *b*2  4*ac*

and , where *b*2

*b*  *b*2  4*ac*

2*a* 2*a*

 4*ac*  0

If b2 – 4ac < 0, then equation does not have real roots.

# Discriminant of a quadratic equation

For the quadratic equation a*x2 + bx + c* = 0, *a*  0, the expression

*b*2  4*ac*

is known as **discriminant**.

1. **Nature of the roots** of a quadratic equation:
2. If *b*2 – 4*ac* > 0, the quadratic equation has **two distinct real roots**.
3. If *b*2 – 4*ac* = 0, the quadratic equation has **two equal real roots**.
4. If *b*2 – 4*ac* < 0, the quadratic equation has **no real roots**.
5. There are many equations which are not in the quadratic form but can be reduced to the quadratic form by simplifications.

# Application of quadratic equations

* + The applications of quadratic equation can be utilized in solving real life problems.
  + Following points can be helpful in solving word problems:

1. Every two digit number ‘*xy*’ where *x* is a ten’s place and *y* is a unit’s place can be expressed

as *xy*  10*x*  *y* .

1. Downstream: It means that the boat is running in the direction of the stream Upstream: It means that the boat is running in the opposite direction of the stream Thus, if

Speed of boat in still water is *x* km/h And the speed of stream is *y* km/h

Then the speed of boat downstream will be *(x + y*) km/h and in upstream it will be (*x − y*) km/h.

1. If a person takes x days to finish a work, then his one day's work = 1

*x*